Published by Institute of Physics Publishing for SISSA

Received: *July 27, 2007*
Accepted: *September 24, 2007*
Published: *October 1, 2007*

Alpha-prime corrections to space-like branes

Shibaji Roy and Harvendra Singh

Saha Institute of Nuclear Physics,
1/AF Bidhannagar, Kolkata 700064, India
E-mail: shibaji.roy@saha.ac.in h.singh@saha.ac.in

Abstract: Space-like branes or S-branes are certain class of time dependent solutions of M/string theories having isometries ISO$(p+1)$ $\times$ SO$(d-p-2,1)$, where $d = 11, 10$ respectively and have singularities at $t = 0$. In [1], we found that the asymptotically flat S-branes have the structure of generalized Kasner geometry near $t = 0$. In this work we evaluate higher order α' corrections perturbatively to the heterotic string Kasner backgrounds to probe the singularity at $t = 0$. We generally find that the perturbative corrections do not permit us to reach the singular point, as the supergravity framework fails near $t \sim \sqrt{\alpha}'$ blurring the origin of space-like singularities. This is analogous to the concept of stretched horizons in the case of black holes.

Keywords: p-branes, Spacetime Singularities.

Contents

1. Introduction

M/String theories are known to admit a variety of time dependent solutions. Space-like or S-branes [2] are certain class of such solutions possessing an isometry ISO$(p + 1) \times$ SO$(9 - p, 1)$ for M-theory and ISO$(p + 1) \times$ SO$(8 - p, 1)$ for string theories. They can be asymptotically non-flat [3] as well as flat [4, 5].[1] The asymptotically flat Sp-branes were constructed in ref. [5] and were found [1] to have a space-like or cosmological singularity at $t = 0$. As time dependent solutions, S-branes are particularly interesting as they might help us to understand the time dependent processes in string theory like brane-antibrane or non-BPS brane decay through rolling tachyon [7−9]. They are also interesting from the cosmological point of view and in particular they are known to give rise to four-dimensional accelerating cosmologies [1, 10−13]. But the acceleration in this case is transient, i.e., it occurs only for a finite interval of time. This feature persists for both asymptotically non-flat and flat solutions [1]. But, because of its transient nature, it does not give rise to high enough e-folding necessary for realistic cosmology. However, the inclusion of higher curvature terms improves the situation and some such studies have been reported in [14, 15] and more recently in [16].

In this paper we will be interested in another aspect of the asymptotically flat S-brane solutions, namely, the cosmological singularity and see the effect of stringy corrections to it. In fact the recent interest in perturbative corrections to string theory has revealed many new insights in the understanding of various aspects of black hole singularities [17−21]. The leading world sheet corrections in some supersymmetric black hole geometries, which violate cosmic censorship and have degenerate horizons with vanishing areas, in fact

[1] The relations between these two classes of S-brane solutions have been discussed in [6].

JHEP10(2007)007

become finite when α' corrections are included. It has been the belief in string theory that generally the space and time singularities must get smoothed out when relevant stringy corrections are taken into account. This has partly been tested in case of (small) black holes where quantum corrections are under control due to the large amount of supersymmetry. However, in non-supersymmetric cases it is rather tricky, as the quantum corrections can be very large and even uncontrolled. But recent study in the case of extremal but non-supersymmetric black holes have led to not very different physics than in the case of supersymmetric ones [22, 23].

As we mentioned our main interest in this work is to study the stringy corrections to the S-brane background particularly near the singular point. But we should keep in mind that the time dependent backgrounds are non-supersymmetric and so the question of perturbative corrections is rather convoluted. The asymptotically flat S-branes were studied previously in [1] and this has provided some interesting new insights, particularly, we found that the near $t = 0$ limit of the S-brane solutions leads to generic Kasner type geometries [24]. Like S-branes, Kasner geometries are inherently singular at $t = 0$ and therefore are interesting cosmological solutions. Also they are much simpler than full S-brane solutions and make a good example for the theoretical understanding of space-like singularities. It would be interesting to study the stringy corrections to these Kasner geometries[2] and see what effect they have on the space-like singularity of this background.

The paper is organised as follows. In section-2 we review the $t \to 0$ limit of S-brane solutions and the resultant generalised Kasner cosmologies. We present the special case of S2-branes. In section-3 we obtain the α' corrections to the Kasner backgrounds in a heterotic string set up. We will consider two cases, namely, in the first case the lowest order dilaton will be taken to be constant while in the second case it will be non-constant. The results are summarised in section-4.

2. Generalized Kasner backgrounds

In [1] the generalized Kasner backgrounds were obtained as the near 'horizon' or $t \to 0$ limit of the S-brane solutions of type II string theory in the lowest order (without α' correction). The ten dimensional action we considered was,

$$S = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 - \frac{e^{a\phi}}{2\cdot n!}F_{[n]}^2\right] \tag{2.1}$$

where κ is related to the ten dimensional Newton's constant, $g = \det(g_{\mu\nu})$, with $g_{\mu\nu}$ being the Einstein-frame metric, R is the scalar curvature, ϕ is the dilaton and $F_{[n]}$ is the n-form field strength. Also a is the dilaton coupling given by $a^2 = 4 - 6(n-1)/(n+2)$ for maximal supergravities. Depending on the value of n we can obtain various Sp-brane (where $p = 8 - n$) solution from the equations of motion following from (2.1) with the appropriate ansatz for the metric and the form-field. We, however, will be looking at S2-brane solution and so we will put $n = 6$. The reason for this is that the S2-brane

[2] Stringy corrections to the bosonic string Kasner backgrounds were studied in [25]. We thank Qasem Exirifird for informing us about this.

has 3-dimensional Euclidean world-volume and so, including the time direction we get the 4-dimensional world whose cosmology we are interested in. Therefore, when dilaton is non-constant $a = -1/2$ and when dilaton is constant, a will be put to zero. Note that $a = 0$ does not correspond to maximal supergravity as we mentioned earlier. The S2-brane solution of the above action has the form [5, 1],

$$\begin{aligned} ds^2 &= F^{\frac{12}{5\chi}} g^{\frac{1}{5}} \left(-\frac{dt^2}{g} + t^2 dH_6^2 \right) + F^{-\frac{4}{\chi}} \sum_{i=1}^{3} (dx^i)^2 \\ e^{2\phi} &= F^{-\frac{32}{5\chi}a} g^{\delta}, \qquad F_{[6]} = b\mathrm{Vol}(H_6) \end{aligned} \tag{2.2}$$

where

$$\begin{aligned} F &= g^{\alpha/2} \cos^2\theta + g^{-\beta/2} \sin^2\theta \\ g &= 1 + 4\omega^5/t^5 \end{aligned} \tag{2.3}$$

In the above, α, β, θ, ω and δ are integration constants and b is the charge parameter. Also H_6 is the 6-dimensional hyperbolic space and dH_6^2 is its line element while $\mathrm{Vol}(H_6)$ is its volume form. Also in the above $\chi = 6 + 8a^2/5$ and so, $\chi = 32/5$ when $a = -1/2$ i.e. the dilaton is non-constant and $\chi = 6$ when dilaton is constant. The paramaters in the solution just mentioned are related as,

$$\begin{aligned} b &= \sqrt{\frac{160}{\chi}} (\alpha + \beta) \omega^5 \sin 2\theta \\ \alpha &= \pm \sqrt{\frac{6\chi - 15\delta^2}{16}} + \frac{a\delta}{2} \\ \beta &= \pm \sqrt{\frac{6\chi - 15\delta^2}{16}} - \frac{a\delta}{2} \end{aligned} \tag{2.4}$$

Now it is easy to check from (2.3) that at early time i.e. as $t \to 0$, the function $F(t)$ behaves as $F(t) \sim t^{-5\alpha/2}$. Note that both the upper and the lower signs of α and β gives exactly same behavior of $F(t)$. So, after some rescaling of coordinates and redefining the time coordinate by $t^{(-3\alpha/\chi)+2} dt \to dt$ we can rewrite the configuration (2.2) as follows,

$$\begin{aligned} ds^2 &= -dt^2 + t^{-\frac{10\alpha}{3(\alpha-\chi)}} \sum_{i=1}^{3} (dx^i)^2 + t^{\frac{(6\alpha-\chi)}{3(\alpha-\chi)}} dR_6^2 \\ e^{2\phi} &\sim t^{\frac{-16a\alpha+5\delta\chi}{3(\alpha-\chi)}} \qquad F_{[6]} = 0 \end{aligned} \tag{2.5}$$

where '$\sim$' means upto a constant which can be absorbed by a constant shift of the dilaton. Note that while obtaining eq.(2.5) from eq.(2.2) we had to replace dH_6^2 by dR_6^2, i.e. the flat space and this is because near $t = 0$, the overall radius of the hyperbolic space becomes very large as can be seen from (2.2) and so effectively this space becomes flat. Also, because of that reason the charge parameter b must vanish identically which implies that in that limit the parameter θ must vanish. The parameter ω can be absorbed into the coordinate rescaling and the redefinition of the dilaton field. So, out of the three independent parameters ω, θ and δ, the solution near $t=0$ is characterized by a single parameter δ (or α which is related to δ by (2.4)). By defining the various exponents of t appearing in the metric and the dilaton in (2.5) as

$$\begin{aligned} ds^2 &= -dt^2 + t^{2p}\sum_{i=1}^{3}(dx^i)^2 + t^{2q}dR_6^2 \\ e^{2\phi} &\sim t^{2\gamma} \end{aligned} \tag{2.6}$$

where

$$p = -\frac{5\alpha}{3(\alpha-\chi)}, \qquad q = \frac{6\alpha-\chi}{6(\alpha-\chi)}, \qquad \gamma = \frac{-16a\alpha + 5\delta\chi}{6(\alpha-\chi)} \tag{2.7}$$

we find that they satisfy,

$$3p + 6q = 1, \qquad \text{and} \qquad 3p^2 + 6q^2 = 1 - \frac{1}{2}\gamma^2 \tag{2.8}$$

This is precisely the generalized Kasner geometry [24] one obtains as the near 'horizon' or near $t=0$ limit of the asymptotically flat S2-brane solution of type II string theory. We remark that in the above a can take only two values, namely, $a=0$ (when dilaton is constant in the lowest order) and $a=-1/2$ (when dilaton is non-constant in the lowest order). Consequently, χ can also take two values, namely, $\chi=6$ (when dilaton is constant) and $\chi=32/5$ (when dilaton is non-constant). Therefore, when dilaton is constant in the lowest order we have $\alpha=\pm 3/2$ and $\gamma=0$. Now since there is no dilaton field the background in this case reduces to Kasner geometry. Putting these values in (2.7) we find that p and q take the following two sets of values,

$$\begin{aligned} &\text{(i) } p = 5/9, \qquad q = -1/9 \\ &\text{(ii) } p = -1/3, \qquad q = 1/3 \end{aligned} \tag{2.9}$$

We will study the stringy corrections to these backgrounds in the next section. Next, we discuss the case when dilaton is non-constant at the lowest order. In this case as we mentioned $a=-1/2$ and $\chi=32/5$. Now since in this case we have two relations (2.8) with three unknowns p, q and γ, there will be infinite number of solutions. We will discuss two simple cases and their stringy corrections in the next section. The first case is

$$(a)\ p = q = 1/9, \qquad \text{and} \qquad \gamma = -4/3 \tag{2.10}$$

and the second case is

$$(b)\ p = \frac{7-8\sqrt{3}}{39}, \qquad q = \frac{3+4\sqrt{3}}{39} \qquad \text{and} \qquad \gamma = -4\left(\frac{3+4\sqrt{3}}{39}\right) \tag{2.11}$$

These are the form of the metric and the dilaton in Einstein frame. But since we have a non-trivial dilaton in the lowest order, the form of the metric will be different in the string frame. We will give the form of the metric in the string frame. The reason behind this is that the α' correction terms are explicitly known [26 – 28] in the string frame. If we write the string frame metric and the dilaton as

$$
\begin{aligned}
ds^2 &= -dt^2 + t^{2p'} \sum_{i=1}^{3} (dx^i)^2 + t^{2q'} dR_6^2 \\
e^{2\phi} &\sim t^{2\gamma'}
\end{aligned}
\tag{2.12}
$$

then the two solutions given above in (a) and (b) take the form in string frame as,

$$
\begin{aligned}
&(\mathrm{i}')\ p' = q' = -1/3, && \text{and} \quad \gamma' = -2 \\
&(\mathrm{ii}')\ p' = -\frac{1}{\sqrt{3}}, \quad q' = 0, && \text{and} \quad \gamma' = -\frac{1}{2}\left(1 + \frac{1}{\sqrt{3}}\right)
\end{aligned}
\tag{2.13}
$$

We will study the stringy corrections to these backgrounds in the next section.

3. Alpha-prime corrections

Although we had discussed S-branes in type II theories in the previous section, here we will restrict ourselves to the special case of heterotic string background. Since the S2-brane we described in (2.5) is chargeless, it is also a solution to the heterotic string theory. The reason for this restriction is that it is the heterotic string theory which contains a non-trivial α' correction term in the form of Gauss-Bonnet term, whereas, in type II string theory the first non-trivial correction comes at α'^3 order [26 – 28] and so the calculation becomes more involved. First we will discuss the case where the dilaton is constant in the lowest order.

3.1 Constant lowest order dilaton and the cosmologies:

We first investigate the special case of heterotic backgrounds where the dilaton is constant in the lowest order.

$$
ds^2 = -dt^2 + A(t)^2 \sum_{i=1}^{3} (dx^i)^2 + B(t)^2 \sum_{j=4}^{9} (dx^j)^2 \tag{3.1}
$$

Note here that the metric has the isometry $R^+ \times \mathrm{SO}(3) \times \mathrm{SO}(6)$ and the time coordinate range $0 \leq t \leq \infty$. The string coupling g_s is taken to be very small, so we shall neglect string loop corrections. One can take the x^j's to be the coordinates on some compact Ricci-flat six-manifold but here we take it to be simple toroidal case. Thus in this solution dilaton does not vary at least in the lowest order, while tensor and gauge fields are switched off. We substitute the above ansatz in the action (which is basically the Einstein-Hilbert action since the dilaton is constant) and then minimise the resulting action with respect to A and B. The metric in (3.1) solves vacuum Einstein equations with

$$
\begin{aligned}
&(\mathrm{i})\ A(t) = t^{5/9}, \qquad B(t) = t^{-1/9} \\
&(\mathrm{ii})\ A(t) = t^{-1/3}, \qquad B(t) = t^{1/3}
\end{aligned}
\tag{3.2}
$$

Note that this is precisely the solution we obtained in cases (i) and (ii) in (2.9). This is not surprising since (2.9) also represents the solutions to the vacuum Einstein equation in ten

JHEP10(2007)007

dimensions (note that these solutions are obtained from (2.5) when the dilaton is constant and the form field is put to zero). However, we note that for both the solutions there is an essential singularity at $t = 0$, where Riemann tensor blows up. The scalar curvature R vanishes identically on-shell, but the contractions like

$$R_{MNPQ}R^{MNPQ} \sim \frac{1}{t^4}$$

blow up at the singularity. Since the latter term appears as the leading world sheet correction in the heterotic string theory it becomes imperative to include them in the string action. However, as soon as we include the higher order α' correction terms it is not true that the dilaton will still remain constant[3] [29]. So, we have to consider the full heterotic string effective action including the dilaton in the leading order of α' and is given by [27, 28]

$$\begin{aligned} S = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\Phi\Big[&R + 4(\partial\phi)^2 \\ &+\frac{\alpha'}{8}\left(R^2_{GB} - 16(G^{MN}\partial_M\phi\partial_N\phi - \nabla^2\phi(\partial\phi)^2 + (\partial\phi)^4)\right)\Big] \end{aligned} \tag{3.3}$$

Here $R^2_{GB} = R_{MNPQ}R^{MNPQ} - 4R_{MN}R^{MN} + R^2$ is the Gauss-Bonnet term, $\Phi \equiv e^{-2\phi}$ is defined for simplicity and $G_{MN} = R_{MN} - g_{MN}(R/2)$ is the Einstein's tensor. Note that (3.2) is still a solution to the above action (3.3) in the leading order. Now the various field equations with α' terms will be solved order by order in the neighborhood of these leading order solutions (i) and (ii). Let us plug the ansatz (3.1) in the above action (3.3) to get,

$$\begin{aligned} S \sim \int dt\Big[&6\Phi\left(AB^6\dot{A}^2 + 5A^3B^4\dot{B}^2 + 6B^5A^2\dot{B}\dot{A}\right) + 6\dot{\Phi}\left(2A^3B^5\dot{B} + A^2B^6\dot{A}\right) + \Phi A^3B^6\frac{\dot{\Phi}^2}{\Phi^2}\Big] \\ +\alpha'\Big[&\Phi\left(15B^2A^3\dot{B}^4 + 60B^3A^2\dot{B}^3\dot{A} + 45B^4A\dot{B}^2\dot{A}^2 + 6B^5\dot{B}\dot{A}^3\right) \\ &+\dot{\Phi}\left(20B^3A^3\dot{B}^3 + B^6\dot{A}^3 + 45B^4A^2\dot{A}\dot{B}^2 + 18B^5A\dot{A}^2\dot{B}\right) \\ &+\frac{1}{2}\Phi\Big\{\left(15A^3B^4\dot{B}^2 + 18A^2B^5\dot{A}\dot{B} + 3AB^6\dot{A}^2\right)\frac{\dot{\Phi}^2}{\Phi^2} \\ &\quad - \frac{1}{2}\left(A^2B^6\dot{A} + 2A^3B^5\dot{B}\right)\frac{\dot{\Phi}^3}{\Phi^3} + \frac{1}{12}A^3B^6\frac{\dot{\Phi}^4}{\Phi^4}\Big\}\Big] \end{aligned} \tag{3.4}$$

Note that the total derivative terms in (3.4) have been dropped since they do not contribute to the equations of motion. It can be noted that leading order terms are quadrartic in time derivatives while next order terms have up to four derivatives in them. The equations of motion for the various fields can be straightforwardly obtained from the variation of the

[3]By the same token one might think that the NSNS field $H_{[3]}$ also can not remain zero once the α' corrections are included. Indeed they can be generated via the gravitational Chern-Simons term. But it can be easily checked that for the Kasner backgrounds the Chern-Simons term is in fact vanishing and so, we can consistently put it to zero. We thank Qasem Exirifard for very useful comments which made us rethink on this.

above action with respect to $A(t)$, $B(t)$ and $\Phi(t)$ and they are given respectively as,

$$\begin{aligned}&\frac{3B^4}{\Phi}\Big(30\Phi^2A^2\dot B^2+12\Phi BA\big(2\Phi\dot B\dot A+A(\dot\Phi\dot B+\Phi\ddot B)\big)+B^2\big(2\Phi^2\dot A^2-A^2(\dot\Phi^2-2\Phi\ddot\Phi)\\&+4\Phi A(\dot\Phi\dot A+\Phi\ddot A)\big)\Big)+\alpha'\Big[\frac{3B^2}{8\Phi^3}\Big(360\Phi^4A^2\dot B^4+480\Phi^3BA\dot B^2\big(2\Phi\dot B\dot A\\&+A(\dot\Phi\dot B+\Phi\ddot B)\big)+B^4\big(A^2(\dot\Phi^4-2\Phi\dot\Phi^2\ddot\Phi)+8\Phi A\dot\Phi(-\dot\Phi^2\dot A+2\Phi\dot A\ddot\Phi+\Phi\dot\Phi\ddot A)\\&+4\Phi^2\dot A(\dot\Phi^2\dot A+2\Phi\dot A\ddot\Phi+4\Phi\dot\Phi\ddot A)\big)+60\Phi^2B^2\dot B\big(6\Phi^2\dot B\dot A^2+A^2(\dot\Phi^2\dot B+2\Phi\dot B\ddot\Phi\\&+4\Phi\dot\Phi\ddot B)+4\Phi A(3\dot\Phi\dot B\dot A+2\Phi\dot A\ddot B+\Phi\dot B\ddot A)\big)+24\Phi B^3\big(A^2\dot\Phi(-\dot\Phi^2\dot B+2\Phi\dot B\ddot\Phi+\Phi\dot\Phi\ddot B)\\&+2\Phi A(\dot\Phi^2\dot B\dot A+2\Phi\dot B\dot A\ddot\Phi+2\Phi\dot\Phi(\dot A\ddot B+\dot B\ddot A))\\&+2\Phi^2\dot A(3\dot\Phi\dot B\dot A+\Phi(\dot A\ddot B+2\dot B\ddot A))\big)\Big)\Big]=0\end{aligned}\tag{3.5}$$

$$\begin{aligned}&\frac{6B^3A}{\Phi}\Big(20\Phi^2A^2\dot B^2+10\Phi BA\big(3\Phi\dot B\dot A+A(\dot\Phi\dot B+\Phi\ddot B)\big)+B^2\big(6\Phi^2\dot A^2-A^2(\dot\Phi^2-2\Phi\ddot\Phi)\\&+6\Phi A(\dot\Phi\dot A+\Phi\ddot A)\big)\Big)+\alpha'\Big[\frac{3B}{4\Phi^3}\Big(120\Phi^4A^3\dot B^4+240\Phi^3BA^2\dot B^2\big(3\Phi\dot B\dot A\\&+A(\dot\Phi\dot B+\Phi\ddot B)\big)+B^4\big(A^3(\dot\Phi^4-2\Phi\dot\Phi^2\ddot\Phi)+24\Phi^3\dot A^2(\dot\Phi\dot A+\Phi\ddot A)+12\Phi A^2\dot\Phi(-\dot\Phi^2\dot A\\&+2\Phi\dot A\ddot\Phi+\Phi\dot\Phi\ddot A)+12\Phi^2A\dot A(\dot\Phi^2\dot A+2\Phi\dot A\ddot\Phi+4\Phi\dot\Phi\ddot A)\big)+40\Phi^2B^2A\dot B\big(18\Phi^2\dot B\dot A^2\\&+A^2(\dot\Phi^2\dot B+2\Phi\dot B\ddot\Phi+4\Phi\dot\Phi\ddot B)+6\Phi A(3\dot\Phi\dot B\dot A+2\Phi\dot A\ddot B+\Phi\dot B\ddot A)\big)+20\Phi B^3\big(6\Phi^3\dot B\dot A^3\\&+A^3\dot\Phi(-\dot\Phi^2\dot B+2\Phi\dot B\ddot\Phi+\Phi\dot\Phi\ddot B)+3\Phi A^2(\dot\Phi^2\dot B\dot A+2\Phi\dot B\dot A\ddot\Phi+2\Phi\dot\Phi(\dot A\ddot B+\dot B\ddot A))\\&+6\Phi^2A\dot A(3\dot\Phi\dot B\dot A+\Phi(\dot A\ddot B+2\dot B\ddot A))\big)\Big)\Big]=0\end{aligned}\tag{3.6}$$

$$\begin{aligned}&\frac{B^4A}{\Phi^2}\Big(30\Phi^2A^2\dot B^2+12\Phi BA\big(3\Phi\dot B\dot A+A(\dot\Phi\dot B+\Phi\ddot B)\big)+B^2\big(6\Phi^2\dot A^2-A^2(\dot\Phi^2-2\Phi\ddot\Phi)\\&+6\Phi A(\dot\Phi\dot A+\dot\Phi\ddot A)\big)\Big)+\alpha'\Big[\frac{B^2}{8\Phi^4}\Big(360\Phi^4A^3\dot B^4+480\Phi^3BA^2\dot B^2\big(3\Phi\dot B\dot A+A(\dot\Phi\dot B\\&+\Phi\ddot B)\big)+B^4\big(A^3(-3\dot\Phi^4+4\Phi\dot\Phi^2\ddot\Phi)+24\Phi^3\dot A^2(\dot\Phi\dot A+\Phi\ddot A)-6\Phi A^2\dot\Phi(-2\dot\Phi^2\dot A+2\Phi\dot A\ddot\Phi.\\&+\Phi\dot\Phi\ddot A)+24\Phi^2A\dot A(-\dot\Phi^2\dot A+\Phi\dot A\ddot\Phi+2\Phi\dot\Phi\ddot A)\big)+120\Phi^2B^2A\dot B\big(9\Phi^2\dot B\dot A^2+A^2(-\dot\Phi^2\dot B\\&+\Phi\dot B\ddot\Phi+2\Phi\dot\Phi\ddot B)+3\Phi A(3\dot\Phi\dot B\dot A+2\Phi\dot A\ddot B+\Phi\dot B\ddot A)\big)+12\Phi B^3\big(12\Phi^3\dot B\dot A^3\\&+A^3\dot\Phi(2\dot\Phi^2\dot B-2\Phi\dot B\ddot\Phi-\Phi\dot\Phi\ddot B)+12\Phi A^2(-\dot\Phi^2\dot B\dot A+\Phi\dot B\dot A\ddot\Phi+\Phi\dot\Phi(\dot A\ddot B+\dot B\ddot A))\\&+12\Phi^2A\dot A(3\dot\Phi\dot B\dot A+\Phi(\dot A\ddot B+2\dot B\ddot A))\big)\Big)\Big]=0\end{aligned}\tag{3.7}$$

The corrected solutions are obtained by taking the perturbative ansatz

$$\begin{aligned} A(t) &= A_0(t) + \alpha' A_1(t) + O(\alpha'^2), \\ B(t) &= B_0(t) + \alpha' B_1(t) + O(\alpha'^2), \\ \Phi(t) &= \Phi_0(t) + \alpha' \Phi_1(t) + O(\alpha'^2) \end{aligned} \tag{3.8}$$

where we take A_0, B_0 and Φ_0 as the known lowest order solutions. Note that in this case the zeroth order dilaton is constant and so, $\Phi_0 =$ constant. We remark that the perturbative ansatz are good only in the time region where $\alpha' A_1(t) \ll A_0(t)$, $\alpha' B_1(t) \ll B_0(t)$ and $\alpha' \Phi_1(t) \ll \Phi_0(t)$. The moment these bounds are violated we will be making wrong move. As we will see that the perturbative ansatz are good particularly in the asymptotic regions.

By substituting (3.8) in (3.5), (3.6) and (3.7) it is easy to see that to zeroth order in α', these equations take the forms

$$\begin{aligned} 15A_0^2\dot{B}_0^2 + 6A_0B_0\left(2\dot{A}_0\dot{B}_0 + A_0\ddot{B}_0\right) + \dot{A}_0^2B_0^2 + 2A_0\ddot{A}_0B_0^2 &= 0 \\ 10A_0^2\dot{B}_0^2 + 5A_0B_0\left(3\dot{A}_0\dot{B}_0 + A_0\ddot{B}_0\right) + 3\dot{A}_0^2B_0^2 + 3A_0\ddot{A}_0B_0^2 &= 0 \\ 5A_0^2\dot{B}_0^2 + 2A_0B_0\left(3\dot{A}_0\dot{B}_0 + A_0\ddot{B}_0\right) + \dot{A}_0^2B_0^2 + A_0\ddot{A}_0B_0^2 &= 0 \end{aligned} \tag{3.9}$$

Case(i). Let us first take the case when $A_0(t) = t^{5/9}$, $B_0(t) = t^{-1/9}$ and $\Phi_0(t) = \Phi_0 =$ constant as in the case (i) which solve (3.9). Now substituting (3.8) with these values of A_0, B_0 and Φ_0 in (3.5), (3.6) and (3.7) we get three equations at the order α' involving $A_1(t)$, $B_1(t)$ and $\Phi_1(t)$ as follows,

$$\begin{aligned} \frac{5}{81}\frac{1}{t^{\frac{32}{9}}} + \frac{1}{t^{\frac{5}{9}}}\left(\frac{4}{3}\dot{\Phi}_1 + 6\dot{B}_1 + 6\dot{A}_1\right) + t^{\frac{4}{9}}\left(3\ddot{\Phi}_1 + 18\ddot{B}_1 + 6\ddot{A}_1\right) &= 0 \\ \frac{35}{81}\frac{1}{t^{\frac{26}{9}}} + t^{\frac{1}{9}}\left(\frac{10}{3}\dot{\Phi}_1 + 15\dot{B}_1 + 15\dot{A}_1\right) + t^{\frac{10}{9}}\left(3\ddot{\Phi}_1 + 15\ddot{B}_1 + 9\ddot{A}_1\right) &= 0 \\ \frac{115}{729}\frac{1}{t^3} + \left(\dot{\Phi}_1 + \frac{16}{3}\dot{B}_1 + \frac{14}{3}\dot{A}_1\right) + t\left(\ddot{\Phi}_1 + 6\ddot{B}_1 + 3\ddot{A}_1\right) &= 0 \end{aligned} \tag{3.10}$$

$A_1(t)$, $B_1(t)$ and $\Phi_1(t)$ can be solved from (3.10) and we find,

$$\begin{aligned} A_1(t) &= -\frac{775}{13122}t^{-\frac{13}{9}}, \\ B_1(t) &= \frac{10}{6561}t^{-\frac{19}{9}}, \\ \Phi_1(t) &= \frac{1}{g_s^2}\frac{115}{1458}t^{-2} \end{aligned} \tag{3.11}$$

Note, as we mentioned earlier, that even though the lowest order Φ i.e. Φ_0 was kept constant, higher order correction makes the dilaton non-trivial. Thus the complete solution up to first order in α' is:

$$A(t) = t^{\frac{5}{9}}\left(1 - \frac{775}{13122}\frac{\alpha'}{t^2}\right), \quad B(t) = t^{-\frac{1}{9}}\left(1 + \frac{10}{6561}\frac{\alpha'}{t^2}\right), \quad \Phi(t) = \frac{1}{g_s^2}\left(1 + \frac{115}{1458}\frac{\alpha'}{t^2}\right) \tag{3.12}$$

JHEP10(2007)007

where we have taken Φ_0 = constant $= 1/g_s^2 \gg 1$. Now the metric and the dilaton becomes,

$$\begin{aligned}
ds^2 &= -dt^2 + t^{\frac{10}{9}}\left(1 - \frac{775}{6561}\frac{\alpha'}{t^2}\right)\sum_{i=1}^{3}(dx^i)^2 + t^{-\frac{2}{9}}\left(1 + \frac{20}{6561}\frac{\alpha'}{t^2}\right)\sum_{j=4}^{9}(dx^j)^2 \\
e^{2\phi} &= g_s^2\left(1 - \frac{115}{1458}\frac{\alpha'}{t^2}\right)
\end{aligned} \tag{3.13}$$

From the above we notice that the α' corrections are such that the metric components involving $A(t)$ identically vanish at some time $t = t_a$. The quantity t_a can be obtained by taking $A(t)^2|_{t=t_a} = 0$ in (3.13), we get

$$t_a = \sqrt{\frac{775}{6561}}\sqrt{\alpha'}\ . \tag{3.14}$$

While the determinant of the metric is

$$\mathrm{Det}(-g) = t^2\left(1 - \frac{245}{729}\frac{\alpha'}{t^2}\right) + O(\alpha'^2)$$

which vanishes for $t = t_g$ where

$$t_g = \sqrt{\frac{245\alpha'}{729}}.$$

Note that we have $t_g > t_a$. Thus the metric becomes degenerate at a finite time interval t_g away from $t = 0$. The $t \gg t_g$ is the region where our perturbative calculation could be trusted. While for $t \leq t_g$ the perturbative analysis will break down. Also, in other words, the physics becomes fuzzy from the supergravity point of view. However we still need to evaluate various curvature quantities to see if those remain finite at $t > t_g$. If that is the case we can say that $t \simeq t_g$ is some kind of a *'horizon of time'* behind that a cosmological singularity is hidden. Some of these quantities are listed below

$$\begin{aligned}
R &= -\frac{230}{243}\frac{\alpha'}{t^4} + O(\alpha'^2) \\
R_{MNPQ}R^{MNPQ} &= \frac{1840}{729t^4} + O(\alpha') \\
R_{MN}R^{MN} &= 0 + O(\alpha') \\
R^2_{GB} &= \frac{1840}{729t^4} + O(\alpha')
\end{aligned} \tag{3.15}$$

By looking at the above expressions we find for $t > t_g$ all these quantities stay finite and well defined. The actual singularity is at $t = 0$ where these quantities will blow up. But, *strictly speaking, these expressions are not valid when* $t \leq t_g$. and we cannot make any firm conclusion on the nature of the singularity. But for $t > t_g$ the spacetime makes perfect sense.

Case(ii). Let us next look at the other solution when $A_0(t) = t^{-1/3}$, $B_0(t) = t^{1/3}$ and Φ_0 = constant as in the case (ii) which also solve (3.9). Again using (3.8) with these values of A_0, B_0 and Φ_0 in (3.5), (3.6) and (3.7) we get three equations at the order α' involving $A_1(t)$, $B_1(t)$ and $\Phi_1(t)$ as follows,

$$
\begin{aligned}
\frac{5}{9}\frac{1}{t^{\frac{8}{3}}} + t^{\frac{1}{3}}\left(4\dot{\Phi}_1 + 30\dot{B}_1 + 6\dot{A}_1\right) + t^{\frac{4}{3}}\left(3\ddot{\Phi}_1 + 18\ddot{B}_1 + 6\ddot{A}_1\right) &= 0 \\
\frac{7}{9}\frac{1}{t^{\frac{10}{3}}} + \frac{1}{t^{\frac{1}{3}}}\left(2\dot{\Phi}_1 + 15\dot{B}_1 + 3\dot{A}_1\right) + t^{\frac{2}{3}}\left(3\ddot{\Phi}_1 + 15\ddot{B}_1 + 9\ddot{A}_1\right) &= 0 \\
\frac{1}{3}\frac{1}{t^3} + \left(\dot{\Phi}_1 + 8\dot{B}_1 + 2\dot{A}_1\right) + t\left(\ddot{\Phi}_1 + 6\ddot{B}_1 + 3\ddot{A}_1\right) &= 0
\end{aligned}
\tag{3.16}
$$

The solutions to the above two equations (3.16) are,

$$
A_1(t) = -\frac{1}{54}t^{-\frac{7}{3}}, \quad B_1(t) = -\frac{1}{27}t^{-\frac{5}{3}}, \quad \Phi_1(t) = \frac{1}{g_s^2}\frac{1}{6t^2} \tag{3.17}
$$

So, the corrected solution upto first order in α' is

$$
A(t) = t^{-\frac{1}{3}}\left(1 - \frac{1}{54}\frac{\alpha'}{t^2}\right), \quad B(t) = t^{\frac{1}{3}}\left(1 - \frac{1}{27}\frac{\alpha'}{t^2}\right), \quad \Phi(t) = \frac{1}{g_s^2}\left(1 + \frac{\alpha'}{6t^2}\right) \tag{3.18}
$$

Here there appears to be more than one horizon. The outer most horizon is obtained by solving $B(t_b)^2 = 0$. It gives

$$
t_b = \sqrt{\frac{2\alpha'}{27}} \tag{3.19}
$$

We obtain the inner horizon at $t = t_a$ where $A(t_a)^2 = 0$. But,

$$
\mathrm{Det}(-g) = t^2\left(1 - \frac{5}{9}\frac{\alpha'}{t^2}\right) + O(\alpha'^2)
$$

and the determinant becomes degenerate for $t = t_g = \sqrt{\frac{5\alpha'}{9}}$ and once again note that $t_g > t_b > t_a$.

The curvature invariants are evaluated as

$$
\begin{aligned}
R &= -2\frac{\alpha'}{t^4} + O(\alpha'^2) \\
R_{MNPQ}R^{MNPQ} &= \frac{16}{3t^4} + O(\alpha') \\
R_{MN}R^{MN} &= 0 + O(\alpha') \\
R^2_{GB} &= \frac{16}{3t^4} + O(\alpha')\ .
\end{aligned}
\tag{3.20}
$$

With α' corrections included the backgrounds are no longer Ricci-flat. But these quantites stay finite for $t > t_g$. So it can be concluded that higher derivative corrections for the simple Kasner geometries are very important in the neighborhood of $t = t_g$. First they resolve the cosmological singularity by hiding the singularity behind the time horizon. It is much like as we find a stretched horizon at string length away from the naked singularity for the BPS black holes which have vanishing horizon area classically. Had we got it differently, we

JHEP10(2007)007

would have been a bit disappointed. The conclusions are naive, that it is meaningless in the low energy supergravity set up to talk of the cosmological events (decay or creation) whose lifespan is smaller than string time $\sqrt{\alpha'}$. Second, in order to discuss or resolve space-like singularities, for example a Big-bang, we need to employ full string theory.

Specially, for both the pure Kasner geometries above we had string coupling fixed at very low value. But for other backgrounds the dilaton will generally vary in time and the string coupling may become very large at $t = 0$.

3.1.1 Spontaneous compactification

It is worth noting that the α' corrected solutions for large t, i.e. when $t \gg \sqrt{\alpha'}$, do become usual Kasner solutions. The sizes and radii of the spatial directions change with time for the Kasner solutions. Particularly in the case (i), for large t the size of the six-dimensional space becomes naturally small and so it can be compactified. We compactify it on T^6 and find that the corresponding 4-dimensional Einstein metric becomes

$$ds^2 = g_s^{-2}\left(-d\tau^2 + a(t)^2\left((dx^1)^2 + (dx^2)^2 + (dx^3)^2\right)\right) \tag{3.21}$$

where $\tau = \frac{3}{2}t^{\frac{2}{3}}(1 - \frac{15}{6561}\frac{\alpha'}{t^2}) + O(\alpha'^2)$ and $a(t) = t^{\frac{2}{9}}(1 - \frac{755}{2\cdot 6561}\frac{\alpha'}{t^2})$. The 4-dimesional dilaton is

$$e^{2\phi_4} = g_s^2 t^{\frac{6}{9}}\left(1 + \frac{20}{6561}\frac{\alpha'}{t^2}\right)^{-3}$$

plus there is volume modulus from compactification. From this we determine that the scale factor

$$a(\tau) = \left(\frac{2}{3}\tau\right)^{1/3}\left(1 - \frac{745}{388}\frac{\alpha'}{\tau^3}\right) + O(\alpha'^2). \tag{3.22}$$

So we can now evaluate the Hubble rate

$$H = \frac{1}{a}\frac{da}{d\tau} = \frac{1}{3\tau}\left(1 + \frac{745}{432}\frac{\alpha'}{\tau^3}\right) + O(\alpha'^2). \tag{3.23}$$

and the deceleration rate

$$\frac{1}{a}\frac{d^2a}{d\tau^2} = -\left(\frac{2}{9} + \frac{3725}{1944}\frac{\alpha'}{\tau^3}\right)\frac{1}{\tau^2} + O(\alpha'^2). \tag{3.24}$$

Since we have horizon at $t = t_g = \sqrt{\frac{245\alpha'}{729}}$ which corresponds to $\tau_g^3 \simeq 1.13\alpha'$. As we see $\ddot{a} < 0$, the cosmological expansion at late times, i.e. for $\tau^3 > \alpha'$, is always decelerating one. From the eq. (3.23) we see that the corrections tend to improve the Hubble rate such that the quantity $\left(\frac{1}{3} + \frac{745}{3\cdot 432}\frac{\alpha'}{\tau^3}\right) \sim .38$ when $\tau^3 = 11.3\alpha'$. Also the deceleration rate is such that it initially decelerates faster than $\frac{2}{9}$ if the corrections are included. In fact we can calculate the corrected deceleration rate as $\left(\frac{2}{9} + (1.92)(.089)\right) \sim 0.24$ which is very close to the value for radiation dominated phase of the universe. It appears as if the universe was decelerating faster, as in the radiation dominated phase, initially.

JHEP10(2007)007

3.2 $O(\alpha')^2$ corrections

It is straightforward to evaluate more higher order corrections to the solutions. Note that the heterotic action does not receive any $O(\alpha'^2)$ corrections for the Kasner background we have chosen [27]. Nevertheless, we can obtain the corrections in the solutions by solving equations to second order in α'. So we take the ansatz

$$\begin{aligned} A(t) &= A_0(t) + \alpha' A_1(t) + \alpha'^2 A_2(t) + O(\alpha'^3), \\ B(t) &= B_0(t) + \alpha' B_1(t) + \alpha'^2 B_2(t) + O(\alpha'^3), \\ \Phi(t) &= \Phi_0(t) + \alpha' \Phi_1(t) + \alpha'^2 \Phi_2(t) + O(\alpha'^3) \end{aligned} \tag{3.25}$$

where we take $A_0, B_0, \Phi_0, A_1, B_1, \Phi_1$ as the known lower order solutions worked out in the previous section. As an illustration we consider the case of the background (i), but similar computation can be done for the background (ii) also. Substituting these ansatze in the equations (3.5), (3.6) and (3.7) and collecting the coefficients of α'^2 terms and equating them to zero, we find three equations involving $A_2(t)$, $B_2(t)$ and $\Phi_2(t)$. Solving those equations we obtain the second order corrections. The complete corrected solutions upto order α'^2 are

$$\begin{aligned} A(t) &= t^{\frac{5}{9}} \left(1 - \frac{775}{13122} \frac{\alpha'}{t^2} - \frac{2125975}{172186884} \frac{\alpha'^2}{t^4}\right) + O(\alpha'^3), \\ B(t) &= t^{-1/9} \left(1 + \frac{10}{6561} \frac{\alpha'}{t^2} + \frac{89975}{172186884} \frac{\alpha'^2}{t^4}\right) + O(\alpha'^3), \\ \Phi(t) &= g_s^{-2} \left(1 + \frac{115}{1458} \frac{\alpha'}{t^2} + \frac{51625}{2125764} \frac{\alpha'^2}{t^4}\right) + O(\alpha'^3) \end{aligned} \tag{3.26}$$

Similarly the curvature and the Gauss-Bonnet term become

$$\begin{aligned} R &= \frac{1}{t^2} \left(-\frac{230}{243} \frac{\alpha'}{t^2} - \frac{482075}{531441} \frac{\alpha'^2}{t^4} + O(\alpha'^3) \right) \\ R^2_{GB} &= \frac{1}{t^4} \left(\frac{1840}{729} + \frac{1320800}{531441} \frac{\alpha'}{t^2} + O(\alpha'^2) \right) \\ R_{MN} R^{MN} &= \frac{68350}{177147} \frac{\alpha'}{t^6} + O(\alpha'^2) \\ R_{MNPQ} R^{MNPQ} &= \frac{1}{t^4} \left(\frac{1840}{729} + \frac{1320800}{531441} \frac{\alpha'}{t^2} + O(\alpha'^2) \right) \end{aligned} \tag{3.27}$$

We can see that this expansion could be arranged as powers of $\frac{\alpha'}{t^2}$. Also the coefficients do not change signs as we go up higher in the order.

Let us also note that the complete on-shell action upto second order corrections can now be expressed as

$$S \sim \int dt \frac{1}{t} \left[\sum_{n=0}^{\infty} a_n \left(\frac{\alpha'}{t^2} \right)^n \right] \tag{3.28}$$

where some of the calculated coefficients are $a_0 = 0,\ a_1 = -\frac{460}{729},\ a_2 = -\frac{287075}{531441}$. The series appears to be convergent.

3.3 Non-constant lowest order dilaton

Having studied the Kasner solutions with the lowest order dilaton taken to be constant in the previous subsections we would like to evalaute the higher derivative corrections to the time-dependent heterotic Sp-brane solutions with non-constant lowest order dilaton field. The complete action upto α' order has been given in (3.3). Using the same metric ansatz in the action we get the same equations of motion (3.5), (3.6) and (3.7). However since in this case the lowest order dilaton is non-constant, the zeroth order equation in α' will involve Φ_0 and its derivatives unlike the similar equations (3.9) in the previous case. Substituting (3.8) in (3.5), (3.6) and (3.7) we get the zeroth order equations in the form,

$$
\begin{aligned}
&30\Phi_0^2 A_0^2 \dot{B}_0^2 + 12\Phi_0 B_0 A_0 \left(2\Phi_0 \dot{B}_0 \dot{A}_0 + A_0 \left(\dot{\Phi}_0 \dot{B}_0 + \Phi_0 \ddot{B}_0\right)\right) \\
&+B_0^2 \left(2\Phi_0^2 \dot{A}_0^2 - A_0^2 \left(\dot{\Phi}_0^2 - 2\Phi_0 \ddot{\Phi}_0\right) + 4\Phi_0 A_0 \left(\dot{\Phi}_0 \dot{A}_0 + \Phi_0 \ddot{A}_0\right)\right) = 0 \\
&20\Phi_0^2 A_0^2 \dot{B}_0^2 + 10\Phi_0 B_0 A_0 \left(3\Phi_0 \dot{B}_0 \dot{A}_0 + A_0 \left(\dot{\Phi}_0 \dot{B}_0 + \Phi_0 \ddot{B}_0\right)\right) \\
&+B_0^2 \left(6\Phi_0^2 \dot{A}_0^2 - A_0^2 \left(\dot{\Phi}_0^2 - 2\Phi_0 \ddot{\Phi}_0\right) + 6\Phi_0 A_0 \left(\dot{\Phi}_0 \dot{A}_0 + \Phi_0 \ddot{A}_0\right)\right) = 0 \\
&30\Phi_0^2 A_0^2 \dot{B}_0^2 + 12\Phi_0 B_0 A_0 \left(3\Phi_0 \dot{B}_0 \dot{A}_0 + A_0 \left(\dot{\Phi}_0 \dot{B}_0 + \Phi_0 \ddot{B}_0\right)\right) \\
&+B_0^2 \left(6\Phi_0^2 \dot{A}_0^2 - A_0^2 \left(\dot{\Phi}_0^2 - 2\Phi_0 \ddot{\Phi}_0\right) + 6\Phi_0 A_0 \left(\dot{\Phi}_0 \dot{A}_0 + \dot{\Phi}_0 \ddot{A}_0\right)\right) = 0
\end{aligned}
\tag{3.29}
$$

Now it can be easily checked that both the solutions given in eqs.(2.12) (2.13) indeed solve the above equations (3.29) as they should. Substituting the first solution (i') of (2.13) in (3.29) we get three equations at the order α' involving $A_1(t)$, $B_1(t)$ and $\Phi_1(t)$ as follows,

$$
\begin{aligned}
&-\frac{6560}{729t^{8/3}} + \frac{88\Phi_1}{t^{14/3}} + \frac{64B_1}{81t^{1/3}} + \frac{64A_1}{243t^{1/3}} - \frac{40\dot{\Phi}_1}{t^{11/3}} + \frac{320}{27}t^{2/3}\dot{B}_1 + \frac{320}{81}t^{2/3}\dot{A}_1 \\
&\qquad + \frac{6\ddot{\Phi}_1}{t^{8/3}} + \frac{64}{9}t^{5/3}\ddot{B}_1 + \frac{64}{27}t^{5/3}\ddot{A}_1 = 0 \\
&\frac{13120}{729t^{8/3}} + \frac{176\Phi_1}{t^{14/3}} + \frac{320B_1}{243t^{1/3}} + \frac{64A_1}{81t^{1/3}} - \frac{80\dot{\Phi}_1}{t^{11/3}} + \frac{1600}{81}t^{2/3}\dot{B}_1 + \frac{320}{27}t^{2/3}\dot{A}_1 \\
&\qquad + \frac{12\ddot{\Phi}_1}{t^{8/3}} + \frac{320}{27}t^{5/3}\ddot{B}_1 + \frac{64}{9}t^{5/3}\ddot{A}_1 = 0 \\
&-\frac{1070}{9t^7} + \frac{162\Phi_1}{t^9} - \frac{567\dot{\Phi}_1}{8t^8} + \frac{16\dot{B}_1}{t^{11/3}} + \frac{8\dot{A}_1}{t^{11/3}} + \frac{81\ddot{\Phi}_1}{8t^7} + \frac{12\ddot{B}_1}{t^{8/3}} + \frac{6\ddot{A}_1}{t^{8/3}} = 0\,.
\end{aligned}
\tag{3.30}
$$

The equations (3.30) can be solved and we find,

$$
A_1(t) = B_1(t) = \frac{755}{54}t^{-\frac{7}{3}} \quad \Phi_1(t) = -\left(\frac{2}{3}\right)^4 \frac{2375}{18}t^2 \tag{3.31}
$$

So, the complete solution upto first order in α' is

$$
A(t) = B(t) = t^{-\frac{1}{3}}\left(1 + \frac{755}{54}\frac{\alpha'}{t^2}\right), \qquad \Phi(t) = \left(\frac{2}{3}t\right)^4 \left(1 - \frac{2375}{18}\frac{\alpha'}{t^2}\right) \tag{3.32}
$$

JHEP10(2007)007

It is interesting to note that the α' corrections do not break the SO(9) invariance of the original solution given in (i') of (2.13). The other covariant quantities of interest are

$$\begin{aligned}
\mathrm{Det}(-g) &= \frac{1}{t^6}\left(1+\frac{755}{3}\frac{\alpha'}{t^2}\right)+O(\alpha'^2)\\
R &= \frac{1}{t^2}\left(16+\frac{28690}{9}\frac{\alpha'}{t^2}\right)+O(\alpha'^2)\\
R_{MNPQ}R^{MNPQ} &= \frac{80}{9t^4}+O(\alpha')\\
R_{MN}R^{MN} &= \frac{32}{t^4}+O(\alpha')\\
R^2_{GB} &= \frac{1232}{9}\frac{1}{t^4}+O(\alpha')\ .
\end{aligned} \tag{3.33}$$

From the determinant of the metric we determine that there is a cut-off time $t=t_g$ below which the calculations cannot be trusted, means perturbative approximation will break down. It is given by

$$(t_g)^2 = \frac{755}{3}\alpha' \tag{3.34}$$

For the time range $t>t_g$ all the curvature expressions stay finite. For large time all the α' corrections to the solution become negligible and the asymptotic generalized Kasner background emerges.

By using the similar technique we can also obtain the corrections of the second solution (ii') of (2.13). We here give the complete solution as,

$$\begin{aligned}
A(t) &= t^{-\frac{1}{\sqrt{3}}}\left\{1+\left(\frac{111+80\sqrt{3}}{72}\right)\frac{\alpha'}{t^2}\right\}\\
B(t) &= 1+\left(\frac{14+9\sqrt{3}}{8}\right)\frac{\alpha'}{t^2}\\
\Phi(t) &= t^{\left(1+\frac{1}{\sqrt{3}}\right)}\left\{1-\left(\frac{365+236\sqrt{3}}{24}\right)\frac{\alpha'}{t^2}\right\}
\end{aligned} \tag{3.35}$$

The other quantities of interest are

$$\begin{aligned}
\mathrm{Det}(-g) &= t^{-2\sqrt{3}}\left\{1+\left(\frac{121(\sqrt{3}+2)}{4\sqrt{3}}\right)\frac{\alpha'}{t^2}\right\}+O(\alpha'^2)\\
R &= \frac{1}{t^2}\left(4+2\sqrt{3}\right)+\left(\frac{1895+1126\sqrt{3}}{6}\right)\frac{\alpha'}{t^4}+O(\alpha'^2)\\
R_{MNPQ}R^{MNPQ} &= \frac{1}{3t^4}\left(20+8\sqrt{3}\right)+O(\alpha')\\
R_{MN}R^{MN} &= \frac{1}{t^4}\left(8+4\sqrt{3}\right)+O(\alpha')\\
R^2_{GB} &= \frac{8}{3t^4}\left(1+\sqrt{3}\right)+O(\alpha')
\end{aligned} \tag{3.36}$$

We again find from the determinant of the metric that the cut-off time in this case is given by

$$t_g^2 = \frac{121(\sqrt{3}+2)}{4\sqrt{3}}\alpha' \tag{3.37}$$

below which the perturbative approximation will break down. However, for $t > t_g$ the curvature invariants remain finite. For large time we again recover the generalized Kasner form of the background.

As indicated in the case of cosmologies with the constant lowest order dilaton studied in the previous subsections, here also the more higher order α' corrections can be computed. The general comments about the spontaneous compactification as well as the improvement on the deceleration rate remain more or less similar in this case also.

4. Summary

In this paper we have studied the α' corrections to a class of time dependent solutions called space-like or S-branes in string theory. S-branes generically have a space-like singularity at $t = 0$. The near 'horizon' or near $t = 0$ limit of these S-branes have the structure of the generalized Kasner geometry which are the global solutions of string theory with singularity at $t = 0$. Since for these solutions curvature blows up at $t = 0$, we have included the higher order curvature terms or the α' correction terms to see their effects on the geometry. We have considered the heterotic string theory Sp-brane solutions since for this case the exact correction terms upto α'^2 order are known. This is in contrary to the type II string theory, where the first non-trivial correction comes at the order of α'^3 and the calculation becomes quite involved. We have used both the constant lowest order dilaton i.e. the usual Kasner like solutions and the non-constant dilaton i.e. the generalized Kasner like solutions. In both cases we found the corrected geometries when the α' corrections are included in the action. The interesting thing is that by obtaining the corrections, we are able to determine the time $t = t_g$ when the curvature starts becoming large for the cases we have studied. For the time $t > t_g$ the spacetime is finite and supergravity is a valid approximation. In the regime $t \leq t_g$ the spacetime curvature becomes higher and string theory is the only valid description there. Although, the Sp-brane backgrounds we have studied exist only for $t > 0$, it will be nevertheless interesting to study those solutions which are explicitly time-reversal symmetric.

While studying spontaneous compactification we find that the Kasner cosmologies are generally decelerating as usual, but the deceleration is faster initially and could be made close to value in the radiation dominated phase of our universe if the higher order corrections are included.

Note added:

After submission of this paper to the Archive we received a very useful correspondence from Qasem Exirifard in which he drew our attention to some of the issues we were not aware of. This has helped us, we hope, to improve our paper.

References

[1] S. Roy and H. Singh, *Space-like branes, accelerating cosmologies and the near 'horizon' limit*, *JHEP* **08** (2006) 024 [hep-th/0606041].

[2] M. Gutperle and A. Strominger, *Spacelike branes*, *JHEP* **04** (2002) 018 [hep-th/0202210].

[3] C.-M. Chen, D.V. Gal'tsov and M. Gutperle, *S-brane solutions in supergravity theories*, *Phys. Rev.* **D 66** (2002) 024043 [hep-th/0204071].

[4] M. Kruczenski, R.C. Myers and A.W. Peet, *Supergravity s-branes*, *JHEP* **05** (2002) 039 [hep-th/0204144].

[5] S. Bhattacharya and S. Roy, *Time dependent supergravity solutions in arbitrary dimensions*, *JHEP* **12** (2003) 015 [hep-th/0309202].

[6] S. Roy, *On supergravity solutions of space-like Dp-branes*, *JHEP* **08** (2002) 025 [hep-th/0205198].

[7] A. Sen, *Rolling tachyon*, *JHEP* **04** (2002) 048 [hep-th/0203211].

[8] A. Sen, *Tachyon matter*, *JHEP* **07** (2002) 065 [hep-th/0203265].

[9] A. Sen, *Field theory of tachyon matter*, *Mod. Phys. Lett.* **A 17** (2002) 1797 [hep-th/0204143].

[10] P.K. Townsend and M.N.R. Wohlfarth, *Accelerating cosmologies from compactification*, *Phys. Rev. Lett.* **91** (2003) 061302 [hep-th/0303097].

[11] N. Ohta, *Accelerating cosmologies from s-branes*, *Phys. Rev. Lett.* **91** (2003) 061303 [hep-th/0303238].

[12] S. Roy, *Accelerating cosmologies from M/string theory compactifications*, *Phys. Lett.* **B 567** (2003) 322 [hep-th/0304084].

[13] N. Ohta, *A study of accelerating cosmologies from superstring/M theories*, *Prog. Theor. Phys.* **110** (2003) 269 [hep-th/0304172].

[14] K.-i. Maeda and N. Ohta, *Inflation from superstring/M theory compactification with higher order corrections. I*, *Phys. Rev.* **D 71** (2005) 063520 [hep-th/0411093].

[15] K. Akune, K.-i. Maeda and N. Ohta, *Inflation from superstring/M-theory compactification with higher order corrections. II: case of quartic Weyl terms*, *Phys. Rev.* **D 73** (2006) 103506 [hep-th/0602242].

[16] G. Niz and N. Turok, *Stringy corrections to a time-dependent background solution of string and M-theory*, *Phys. Rev.* **D 75** (2007) 126004 [arXiv:0704.1727].

[17] A. Dabholkar, *Exact counting of black hole microstates*, *Phys. Rev. Lett.* **94** (2005) 241301 [hep-th/0409148].

[18] A. Dabholkar, R. Kallosh and A. Maloney, *A stringy cloak for a classical singularity*, *JHEP* **12** (2004) 059 [hep-th/0410076].

[19] A. Sen, *How does a fundamental string stretch its horizon?*, *JHEP* **05** (2005) 059 [hep-th/0411255].

[20] A. Sen, *Black holes, elementary strings and holomorphic anomaly*, *JHEP* **07** (2005) 063 [hep-th/0502126].

[21] A. Sen, *Black hole entropy function and the attractor mechanism in higher derivative gravity*, *JHEP* **09** (2005) 038 [hep-th/0506177].

[22] A. Dabholkar, A. Sen and S.P. Trivedi, *Black hole microstates and attractor without supersymmetry*, *JHEP* **01** (2007) 096 [hep-th/0611143].

[23] D. Astefanesei, K. Goldstein and S. Mahapatra, *Moduli and (un)attractor black hole thermodynamics*, hep-th/0611140.

[24] S.R. Das, J. Michelson, K. Narayan and S.P. Trivedi, *Time dependent cosmologies and their duals*, *Phys. Rev.* **D 74** (2006) 026002 [hep-th/0602107].

[25] G. Exirifard and M. O'Loughlin, *Two and three loop α' corrections to T-duality: Kasner and Schwarzschil*, *JHEP* **12** (2004) 023 [hep-th/0408200].

[26] M.D. Freeman, C.N. Pope, M.F. Sohnius and K.S. Stelle, *Higher order σ-model counterterms and the effective action for superstrings*, *Phys. Lett.* **B 178** (1986) 199;
M.T. Grisaru, A.E.M. van de Ven and D. Zanon, *Four loop β-function for the $N = 1$ and $N = 2$ supersymmetric nonlinear σ-model in two-dimensions*, *Phys. Lett.* **B 173** (1986) 423;
Q.-H. Park and D. Zanon, *More on σ-model β-functions and low-energy effective actions*, *Phys. Rev.* **D 35** (1987) 4038;
I. Jack, D.R.T. Jones and D.A. Ross, *On the relationship between string low-energy effective actions and $O(\alpha'^3)$ σ-model β-functions*, *Nucl. Phys.* **B 307** (1988) 130.

[27] D.J. Gross and J.H. Sloan, *The quartic effective action for the heterotic string*, *Nucl. Phys.* **B 291** (1987) 41.

[28] C.M. Hull and P.K. Townsend, *String effective actions from σ-model conformal anomalies*, *Nucl. Phys.* **B 301** (1988) 197.

[29] C.G. Callan Jr., R.C. Myers and M.J. Perry, *Black holes in string theory*, *Nucl. Phys.* **B 311** (1989) 673.

JHEP10(2007)007